

SECOND SEMESTER EXAMINATION, 2007-08

MECHANICAL ENGINEERING

Time : 3 Hours]

[Total Marks : 100

- Note :** (1) Attempt all questions.
 (2) All questions carry equal marks.
 (3) Be precise in your answer.
 (4) No **second** answer book will be provided.

Q. 1. Attempt and four parts of the following : **5 × 4 = 20**

(a) Explain with examples, what are microscopic and macroscopic point of views to study the subject to thermodynamics.

Ans. From microscopic point of view a system can be considered comprising of numbers of molecules in different state of energies, velocities and occupying various position. However there state is changing continuously due to collision or by the force at a distance. The behaviour of the system is predicted by averaging. It is also a statistical approach of static thermodynamics. Examples is study of Kinetic theory of gases.

Macroscopic or total point of view as general treated in classical thermodynamics of the substance. These properties represent the average value of the system represented by the quantities like pressure, volume, temperature, mass etc, which can be measured and preserved into system.

Examples are study of Compressor, heat engines, turbines, pumps.

Q. 1. (b) How will you define temperature? A metal block of 5 kg and temperature 200°C is submerged into water whose mass is 8 kg and temperature is 30°C. If

the specific heat of metal is 0.2 kJ/kgK, what will be the final temperature of the system?

Ans. Temperature is a property of the system on thermal state of body which distinguishes a hot body to a cold body.

Given data :

$$m_w = 8 \text{ Kg } T_w = 30^\circ\text{C},$$

$$T_b = 200^\circ\text{C } C_b = 0.2 \text{ KJ/Kg.K}$$

$$m_b C_b (T_b - T_f) = m_w C_w (T_f - T_w)$$

$$5 \times 0.2 (200 - T_f) = 8 \times 4.2 (T_f - 30)$$

$\therefore T_f = 34.91 \approx 35^\circ\text{C}$ is the final temperature of the system.

Q. 1. (c) Air during a reversible process is compressed from initial pressure 12 kN/m² to 6 times the initial pressure. Due to this compression volume of air decreases from initial volume 4 m³ to 1.8 m³. Calculate:

(i) Law of the process

(ii) Work done in compressing the air

Ans. $P_i = 12 \text{ kN/m}^2,$

$$P_f = (6 \times 12) = 72 \text{ kN/m}^2$$

$$v_i = 4 \text{ m}^3, V_f = 1.8 \text{ m}^3$$

$$P_i v_i^n = P_f v_f^n$$

$$\therefore 12 \times (4)^n = (72)(1.8)^n$$

$$\therefore \left(\frac{4}{1.8}\right)^n = \left(\frac{72}{12}\right) \approx 6$$

$\therefore n = 2.24$ So the process is polytropic.

$$W = \frac{P_i V_i - P_f V_f}{n - 1}$$

$$= \frac{(12 \times 4 - 72 \times 18)}{(224 - 1)} = -64.25 \text{ KN}$$

So, the process is compression.

Q. 1. (d) Two carnot refrigerators A and B are arranged in series. Obtain the COP of thin composit system in terms of COP of refrigerator A and B only.

Ans. $[C.O.P.]_A = \frac{T_1}{T_2 - T_1} = R_A \quad \dots(1)$

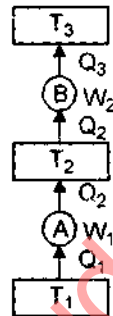
$$[C.O.P.]_B = \frac{T_2}{T_3 - T_2} = R_B \quad \dots(2)$$

$$[C.O.P.]_C = \frac{T_1}{T_3 - T_1} = R_C \text{ (C.O.P}$$

for composite system) $\dots(3)$

From equ. (1) $T_2 R_A - T_1 R_A = T_1$

$$\therefore T_2 = \left[\frac{1 + R_A}{R_A} \right] T_1 \quad \dots(4)$$



Substituting this value (T_2) from equation (4) in equation (2) we get

$$R_B = \frac{\left(\frac{1 + R_A}{R_A} \right) T_1}{T_3 - \left(\frac{1 + R_A}{R_A} \right) T_1} = \frac{(1 + R_A) T_1}{T_3 (R_A) - (1 + R_A) T_1}$$

$$\therefore T_3 (R_A) (R_B) - (1 + R_A) (R_B) T_1 = (1 + R_A) T_1$$

$$\therefore T_3 = T_1 \frac{[(1 + R_A)(1 + R_B)]}{R_A - R_B}$$

$$R_C = \frac{T_1}{T_1(1 + R_A)(1 + R_B)} \cdot T_1$$

$$\therefore \frac{R_A \cdot R_B}{(1 + R_A)(1 + R_B) - R_A \cdot R_B}$$

$$R_C = \frac{R_A \cdot R_B}{1 + R_A + R_B}$$

Q. 1. (e) What is Carnot theorems? What are its different corrolaries? Explain.

Ans. Carnot theorem : No heat engine operating on a cycle between two heat reservoir at different fixed temperature can be more efficient than a reversible engine (like Carnot engine).

Corrolarie one : Two reversible heat engines operating between two heat reservoir at different fixed temperature will have the same efficiency.

Corrolarie two : The efficiency of a reversible engine is independent of the nature or amount of the working substance undergoing the cycle.

Q. 1. (f) Block diagrams of two systems are given below. Giving proper reasons indicate

(i) Name of the system (i.e; HE, RE or HP)

(ii) Type of cycle is possible or impossible and reversible or irreversible.

Ans. Both of the systems heat is flown from higher temperature to power temperature. So this system are heat engine.

$$\text{In system (i) } \eta = \frac{W}{Q} = \frac{700}{1000} = 0.7$$

Carnot efficiency

$$(\eta_c) = \frac{T_1 - T_2}{T_1} = \frac{(500) - (27 + 273)}{500} = 0.4$$

as $\eta_1 \neq \eta_c$ so cycle is irreversible but possible.

$$(ii) \text{ For 2nd system } \eta = \frac{W_2}{Q_2} = \frac{600}{1000} = 0.6$$

Carnot efficiency for 2nd system

$$\eta_C = \frac{T_1 - T_2}{T_1} = \frac{900 - (30 + 273)}{900} = 0.6$$

as $\eta_2 = \eta_C$ so cycle is possible and system is reversible.

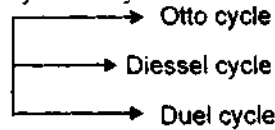
Q. 2. Attempt any two parts :

10 × 2 = 20

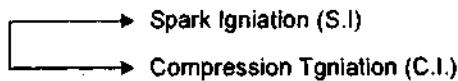
(a) (i) What are different types of IC engine? Why the compression ratio in a CI engine is greater than for a SI engine, explain?

Ans. Different types of I.C engines are

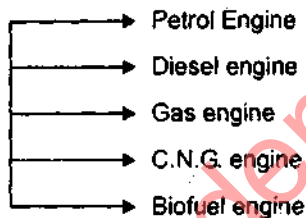
Thermodynamic cycle base



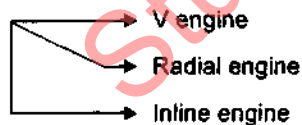
Tgnition system base



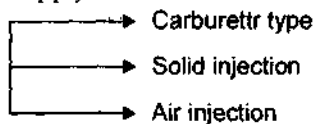
Fuel used



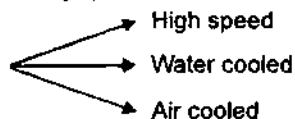
Arrangement of cylinder



Fuel Supply



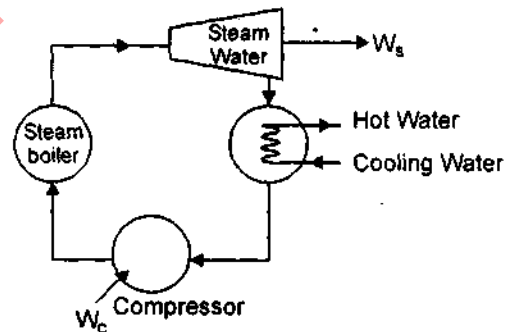
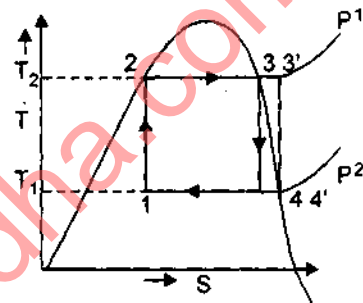
Cooling system



In C.I engine spark plug is not required, it fuel is combusted by compression only. To achieve higher heat in compression process it requires larger compression ratio than spark ignition (S.I) engine.

(ii) Sketch a carnot cycle for water-steam system. Why is Carnot cycle not used as thermodynamic cycle for the steam power plant?

(ii) Carnot cycle for water



Through the carnot cycle is most efficient cycle between the specified temperature limit, however the following inharent practical difficulties make the carnot cycle uneconomical for producing power.

(i) The condenser process (4 - 1) has to be controlled accurately to end up with the desired quality of steam at state 1 which is difficult to achieve in practice.

(ii) Net work done per cycle will be small i.e., it has very low work ratio.

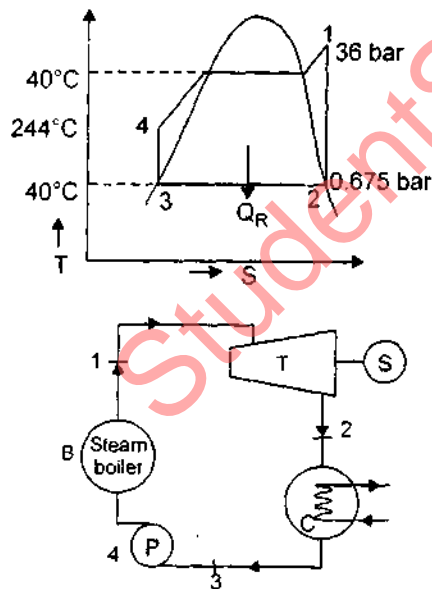
(iii) Isothermal heating at constant pressure is process (2-3) in superheated region is extremely difficult.

(iv) The heat transferred to the cycle is normally obtained by combustion process in a furnace, the product of combustion can be solved theoretically upto the lowest temperature of the cycle. But practically it is not possible.

Q. 2. (b) In a steam power plant, steam is supplied to the turbine at 36 bar and 410°C. The condenser pressure is 0.075 bar. If the turbine develops a power of 12MW calculate for a theoretical cycle:

- (i) Mass flow rate of steam
- (ii) Heat addition and heat rejection
- (iii) Pump work
- (iv) Thermal efficiency

Ans. From Steam table



From steam table

$$P_1 = 36 \text{ bar } T_{sat} = 294.2^\circ\text{C}$$

$$T_1 = 410^\circ\text{C } S_1 = 6.853 \text{ KJ/Kg}$$

$$P_2 = 0.075 \text{ bar (saturation point)}$$

$$T_2 = 40.32^\circ\text{C}$$

$$h_{f2} = 168.8 \text{ KJ/Kg}$$

$$h_{fg2} = 240.4 \text{ KJ/Kg}$$

$$h_{g2} = 2574.8 \text{ KJ/Kg}$$

$$S_{f2} = S_3 = 0.576 \text{ KJ/Kg}$$

$$S_{g2} = S_1 = 8.259 \text{ KJ/KgK}$$

$$S_1 = S_2 = S_{f2} + xS_{fg2}$$

$$x_2 = 0.818 = 0.82$$

$$h_2 = hf_2 + x_2 h_{fg2}$$

$$= 168.8 + 0.818 (2406.0)$$

$$= 2136.92 \text{ KJ/Kg}$$

$$\text{Work done} = m_s(h_1 - h_2)$$

$$12 \times 10^3 = m_s(3250 - 2136.92)$$

$$\therefore m_s = 10.78 \text{ KJ/Kg}$$

$$(i) \therefore \text{mass flow rate of the system} = 10.78$$

KJ/Kg

(ii) Heat supplied

$$(Q_S) = T_{sat}(S_1 - S_4) = 517.2 (6.853 - 0.576)$$

$$= 3246.46 \text{ KJ/Kg}$$

$$= 10.78 \times 3246.36 \text{ KJ}$$

$$= 34.99 \text{ MJ}$$

$$\text{Heat rejected } (Q_R) = T_R(S_2 - S_3)$$

$$= 313.3 (6.855 - 0.576) = 1967.21 \text{ KJ/Kg}$$

$$= 21.20 \text{ Mw}$$

(iii) Total pump work

$$(W_P) = Q_S - Q_R = 34.99 - 21.2 = 1.79 \text{ Mw}$$

$$\text{Efficiency } \eta_R = \frac{\text{Total work done}}{\text{Heat supplied}}$$

$$= \frac{Q_S - Q_R}{Q_S} = \frac{34.99 - 21.2}{34.99}$$

$$= 0.399$$

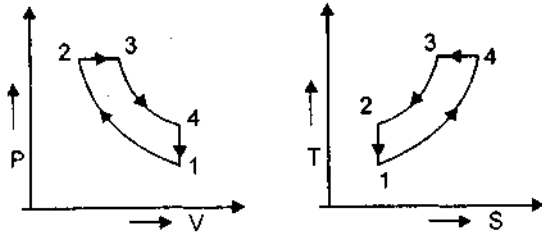
$$= 39.4\%$$

Q. 2. (c) For a diesel cycle following data were observed. Air inlet pressure and temperature = 1.01 bar and 300 K
Compression ratio = 20

Cut off ratio = 2

Calculate the temperatures at all points of the cycle, net power output and thermal efficiency of the cycle.

Ans.



$$T_1 = 300 \text{ K}, P_1 = 1.01 \text{ bar } r = 20, \rho = 2 \quad r = \left(\frac{V_1}{V_2} \right)$$

$$T_2 = T_1 r^{\gamma-1} = 300 (20)^{1.4-1} = 994.34^\circ \text{ K}$$

$$T_3 = T_2 \rho = 994.34 \times 2 = 1988.68^\circ \text{ K}$$

$$T_4 = \rho^{\gamma} T_1 = (2)^{1.4} \times 300 = 791.70^\circ \text{ K}$$

$$\eta_{th} = 1 - \frac{1}{r^{\gamma-1}} \left(\frac{\rho^{\gamma} - 1}{\rho - 1} \right)$$

$$= 1 - \frac{1}{(1.4)(20)^4} \frac{(2^{1.4} - 1)}{(2 - 1)} = 0.65$$

$$= 65\%$$

Net power output (P)

$$= m C_p (T_3 - T_2) - m C_v (T_4 - T_1)$$

$$= 1005 \times (1988.68 - 994.34) - 0.718 (791.70 - 300)$$

$$= 646.27 \text{ kW}$$

Q. 3. Attempt any two parts of the following: $10 \times 2 = 20$

(a) Explain the following :

(i) General condition of equilibrium of a system of coplanar concurrent forces.

(ii) Moment of a couple. Show that a force acting at a point is equivalent to a force couple system at another point.

(iii) Laws of dry friction.

(iv) Belt friction and its applications.

Ans. For equilibrium of a system of coplanar concurrent force

$$(i) \Sigma f_x = 0 \text{ \& } \Sigma f_y = 0$$

If three forces are there Lami's equation is used.

(ii) When two equal unlike parallel non collinear forces act on a body it forms a couple. As a result of applied couple the body starts rotating about its axis of rotation. In such case resultant is zero due to equal and opposite forces acting on a body.

$$R = F - F$$

$$R = 0$$

a = arm of couple

$$F \times a = \text{couple}$$

(iii) Laws of dry friction :

(a) The frictional force always acts tangential at the common surface in contact between the two bodies.

(b) The frictional force always acts in opposite direction to the direction of the applied force or to the direction of motion of the body.

(c) The force of friction can be increased up to limiting friction.

(d) When the body is just on the point of motion, it is in limiting equilibrium.

(e) When the body is in limiting equilibrium, then ratio of limiting friction and normal reaction is constant which is termed as coefficient of friction (μ).

(i) The static friction is always more than the kinetic friction.

(ii) The coefficient of friction only depends upon the nature of surfaces in contact and is independent of weight of a body and area of the surface of contact.

(iii) Frictional force is a self-adjusting force to the applied force but varying up to the limiting value of friction.

(iv) The transmission of power by belt or rope drives depends on the frictional resistance developed between belt and driving surfaces. If T_1 is in tight side tension and T_2 is in slack side tension $\frac{T_1}{T_2} = e^{\mu\theta}$ where θ is lap angle.

Belt friction is use in pulley and rope arrangement driving machinery from motor.

Q. 3. (b) Forces 7, 1, 1 and kN at one of the angular points of a regular pentagon towards four other angular points taken in order. Obtain the resultant of this force system. What is its direction?

Ans.

$$R \cos \theta = 7 \cos 0^\circ$$

$$+ 1 \cos 36^\circ + 1 \cos 70^\circ + 3 \cos 108^\circ$$

$$= 7.191 \text{ km}$$

$$R \sin \theta = 7 \sin 0^\circ + 1 \sin 36^\circ + 1 \sin 72^\circ + 3 \sin 108^\circ$$

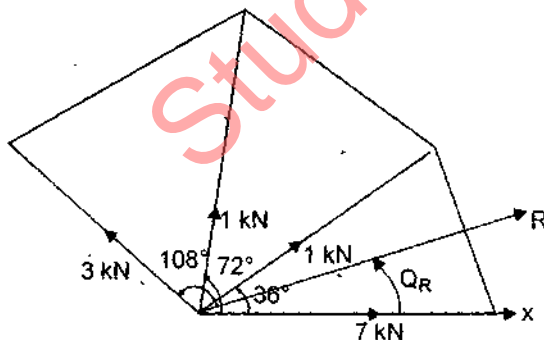
$$= 4.312 \text{ km}$$

$$R = \sqrt{(R \cos \theta)^2 + (R \sin \theta)^2}$$

$$= \sqrt{(7.19)^2 + (4.312)^2}$$

$$= 7.959 \text{ km}$$

$$\theta_R = \tan^{-1} \left(\frac{4.312}{7.191} \right) = 312^\circ$$

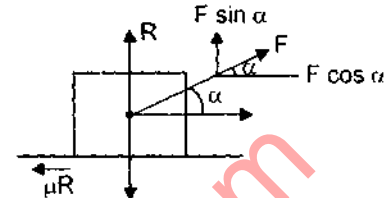


Its direction is 312° with 7 kN component.

Q. 3. (c) A block of stone weighing 50 kN rests on a horizontal floor. If the coefficient of friction between floor and block is 0.3, and if a man pulls the block through a

string which makes an angle α with the horizontal; find for what value of the force necessary to move the block will be minimum. Find this force also.

Ans.



$$F \sin \alpha + \frac{F \cos \alpha}{\mu} = 50$$

$$\therefore F \left(\sin \alpha + \frac{\cos \alpha}{0.3} \right) = 50$$

For minimum value of force

$$\frac{d}{d\alpha} \left\{ F \left(\sin \alpha + \frac{\cos \alpha}{\mu} \right) \right\} = 0$$

$$\therefore \mu \cos \alpha = \sin \alpha$$

$$\therefore \mu = \tan \alpha = 0.3$$

$$\therefore \alpha = 16.69^\circ$$

$$\therefore F = \frac{50 \times 0.3}{0.086 + 0.957} = 14.38 \text{ kN}$$

is the minimum force necessary to move the block.

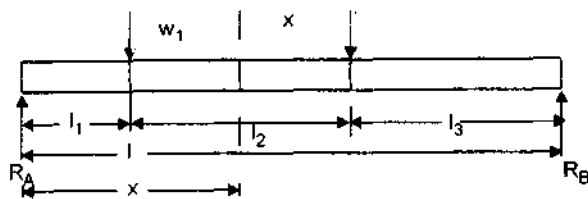
Q. 4. Attempt any two parts of the following : 10 × 2 = 20

(a) (i) Define a beam. Explain how shear force and bending moments are developed at different sections of the beam.

(ii) How are the trusses classified? What are the assumptions taken while analysing a plane. truse ?

Ans. (i) Beam : It is a structural member which is primarily subjected to a system of external loads that act transverse to its axis.

The shear force at any section of beam is the algebraic sum of all the vertical forces acting either to the left or right of section.



Consider a simply supported beam AB having two concentrated load W_1 and W_2 as shown in figure.

By definition of S.F at any section is the algebraic sum of vertical forces to the left of section. Therefore $F_x = R_A - W_1$

It is also equal to the sum of forces to right of section

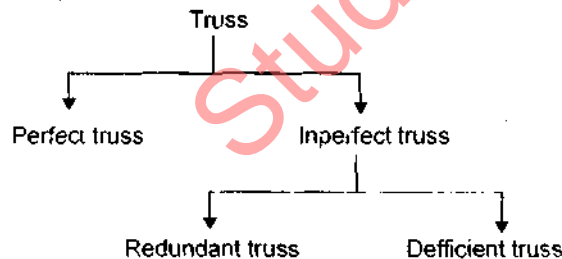
$$\begin{aligned} F_x &= -R_B + W_2 \\ &= -[(W_1 + W_2) - R_A] + W_2 \\ &= R_A - W_1 \end{aligned}$$

This way the shear force at various section of the beam can be calculated.

Bending moment at any section of the beam is the algebraic sum of moment of all the vertical forces acting either to the left or right of the section.

$$M_x = R_A \cdot x - W_1 (x - l_1)$$

(ii) Trusses are classified in following categories.



Perfect truss (stable truss) :

A truss which does not collapse under the loading is called stable truss. In this truss $n = 2j - 3$ is satisfied where j = number of joints, n = number of members,

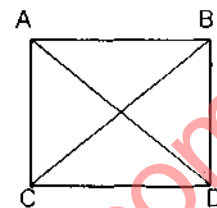
Example is triangular truss.

Imperfect truss (Unstable truss) :

A truss which collapse when loaded is called unstable truss. In unstable truss the condition $n = 2j - 3$ is not satisfied.

Redundant truss :

A truss which $n > 2j - 3$, the truss is called redundant truss in figure $n = 6, j = 4$

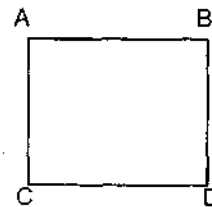


$$n > 2j - 3$$

Deficient truss :

A truss in which $n < 2j - 3$ the truss is called as deficient truss in figure $j = 4, n = 4$

$$n < 2j - 3$$



Assumption taken while analysing a plane truss :

(i) This joints of a truss are assumed to be pin connected and friction less so cannot resist moments.

(ii) The truss is a perfect truss.

(iii) The truss is loaded at the joints only.

(iv) The members of the truss are straight, uniform and they are two force members.

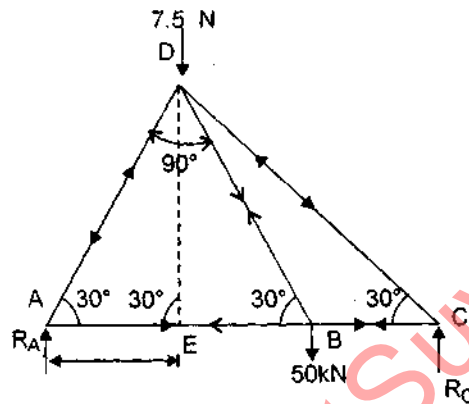
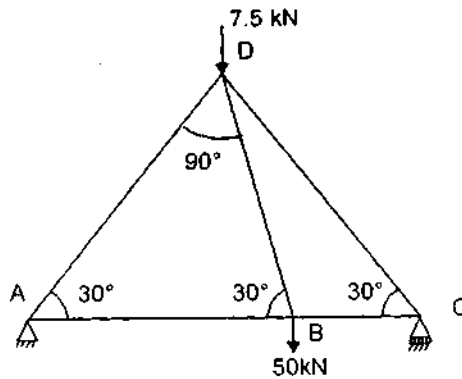
(v) Weight of members are neglected.

Q. 4. (b) Determine the forces and their nature in each member of the truss loaded as shown in Fig 1.

Ans.

$$\sum F_v = 0 \therefore -7.5 + R_A + R_C - 50 = 0$$

$$\therefore R_A + R_C = 57.5 \text{ kN} \quad \dots(i)$$



Taking moment about C =

$$R_A \times 2a - 7.5 \left(a + \frac{a}{3} \right) - 50 \left(a - \frac{a}{3} \right) = 0$$

$$\therefore R_A = 21.33 \text{ kN}$$

$$R_C = 36.17 \text{ kN}$$

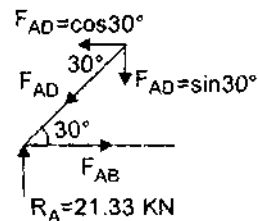
At joint A

$$\therefore F_{AD} \sin 30^\circ = 21.33$$

$$F_{AD} = \frac{21.33}{\sin 30^\circ} = 42.66 \text{ kN (C)}$$

$$F_{AB} = F_{AD} \cos 30^\circ$$

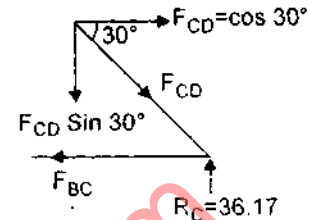
$$= 42.66 \cos 30^\circ$$



$$= 36.94 \text{ kN (T)}$$

In Joint C

$$F_{CD} \sin 30^\circ = R_C = 36.17$$

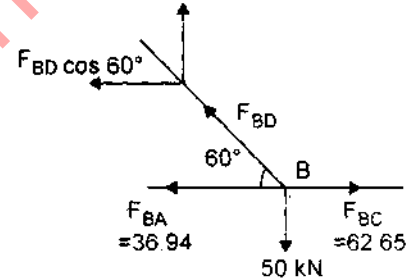


$$F_{CD} = 72.34 \text{ kN (C)}$$

$$F_{BC} = F_{CD} \cos 30^\circ = 62.65 \text{ kN (I)}$$

In joint B

$$\therefore F_{BC} - F_{BA} - F_{BD} \cos 60^\circ = 0$$



$$F_{BD} \cos 60^\circ = F_{BC} - F_{BA}$$

$$F_{BD} = \frac{62.65 - 36.94}{\cos 60^\circ} = 51.42 \text{ kN (T)}$$

Force	Magnitude	Nature
F_{AB}	36.94 kN	T
F_{AD}	42.66 kN	C
F_{CD}	72.34 kN	C
F_{BC}	62.65 kN	I
F_{BD}	51.42 kN	I

Q. 4. (c) Draw the shear forces and bending moment diagrams for the beam shown in Figure 2.

Ans. $R_A + R_B = 20 + 20 + 20 + 4 + 4 = 68$
kN.

$$\Sigma M_A = 0$$

$$\therefore R_A \times 0 - 20 \times 1 - 4$$

$$\times 2 - 20 \times 3 - 4 \times 4 - 20 \times 5 + R_B \times 6 = 0$$

$$R_B = 34 \text{ kN}, R_A = 34 \text{ kN}$$

Shear force between AC = 34 kN

Shear force between GC = $34 - 20 = 14$ kN

Shear force between CD = $14 - 4 = 10$ kN

Shear force between

$$DE = 10 - 20 = -10 \text{ kN}$$

Shear force between

$$EF = -10 - 4 = -14 \text{ kN}$$

Shear force between

$$FB = -14 - 20 = -34 \text{ kN}$$

$$\text{at } B = -34 + 34 = 0$$

Bending moment Calculation :

Bending moment at point A = 0

$$\text{Bending moment at point } C = 34 \times 2 - 20 \times 1 = 48 \text{ kN-m}$$

$$\text{Bending moment at point } D = 34 \times 3 - 20 \times 2 - 4 \times 1 = 58 \text{ kN-m}$$

$$\text{Bending moment at point } E = 34 \times 4 - 20 \times 3 - 4 \times 2 - 20 \times 1 = 48 \text{ kN-m}$$

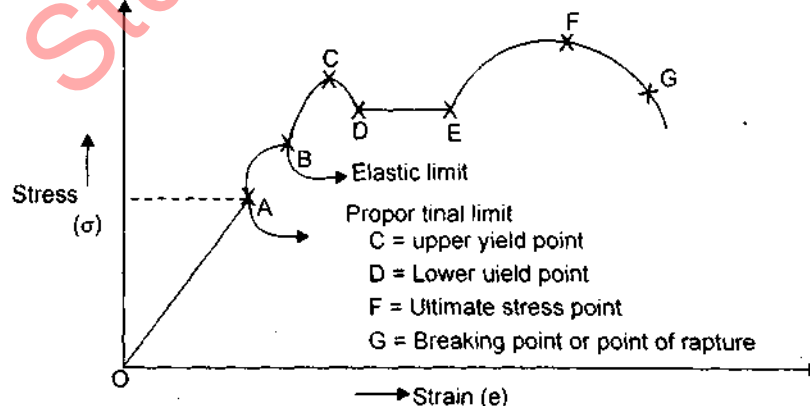
$$\text{Bending moment at point } B = 34 \times 6 - 20 \times 5 - 4 \times 4 - 20 \times 3 - 4 \times 2 - 20 \times 1 = 0 \text{ kN-m}$$

Q. 5. Attempt any four parts of the following :

5 × 4 = 20

(a) Draw stress-strain diagram for a ductile material and define different points shown on it.

Ans.

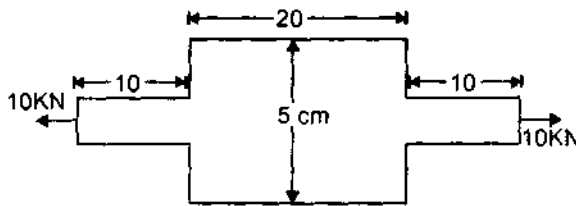


Q. 5. (b) A round bar 40 cm long has 5 cm diameter for middle half of its length and a reduced diameter at the two ends (ends are equal in diameter and length-wise) Bar carries axial

load of 10 kN. Find the diameter and end section if the total allowable extension is 0.03 cm

$$E = 200 \text{ GN/m}^2.$$

Ans.



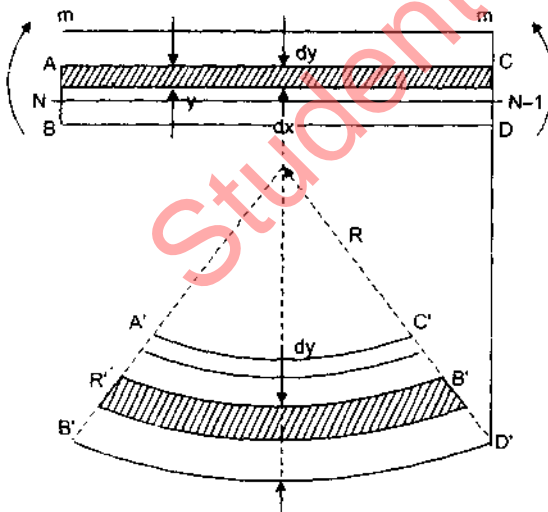
$$\delta l = 0.03 \text{ cm} = 0.03 \times 10^{-2} \text{ m}$$

$$\frac{\rho l_1}{A_1 E} + \frac{\rho l_2}{A_2 E} + \frac{\rho l_3}{A_3 E} = 0.03 \times 10^{-2}$$

$$= \frac{10 \times 10^{-3}}{200 \times 10^9} \left[\frac{2 \times 10 \times 10^{-2} \times 4}{z d^2} + \frac{20 \times 10^{-2} \times 4}{z \times (5)^2 \times 10^{-4}} \right]$$

$$= 0.03 \times 10^{-2}$$

$$\therefore d = 6.57 \text{ cm}$$



Q. 5. (c) Calculate the value of principal stresses and the planes on which they occur for the stresses shown in Figure 3.

Also calculate the plane on which maximum shear stresses are occurring. What are the values of normal stresses on these planes.

$$\text{Ans. } \sigma_x = 100 \text{ MN/m}^2 \quad \sigma_y = 50 \text{ MN/m}^2$$

$$\tau_{xy} = 40 \text{ MN/m}^2$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

$$= \frac{100 + 50}{2} \pm \sqrt{\left(\frac{100 - 50}{2} \right)^2 + 40^2}$$

$$\sigma_1 = 122.16 \text{ MN/m}^2$$

$$\sigma_2 = 27.84 \text{ MN/m}^2$$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$= \frac{122.16 - 27.84}{2} = 47.16 \text{ kN/m}^2$$

$$\tan(2\theta) = \frac{2\tau}{\sigma_x - \sigma_y} = \frac{2 \times 40}{100 - 50} = \frac{80}{50} = 1.6 \therefore$$

$$2\theta = 57.99^\circ, \theta = 28.99^\circ$$

$$\theta_2 = 90 + 28.99 = 118.99^\circ$$

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos^2 \theta$$

$$= \frac{100 + 50}{2} + \frac{100 - 50}{2} \cos^2 45^\circ$$

$$= 75 \text{ kN/m}^2$$

Q. 5. (d) Derive the simple bending equation

$$\frac{N}{I} = \frac{T_b}{Y} = \frac{E}{R}$$

Also mention the assumptions made in the derivation.

Ans. Length of layer PQ after applying moment = P'Q'

$$\text{Decrease in length } PQ = PQ - P'Q'$$

$$\text{Strain } \Sigma = \frac{PQ - P'Q'}{PQ}$$

Also from geometry of the curved beam $P'Q'$ and $N'N$ are similar

$$\frac{P'Q'}{NN'} = \frac{R-y}{R}$$

$$1 - \frac{P'Q'}{NN'} = 1 - \left(\frac{R-y}{R} \right)$$

$$\frac{NN' - P'Q'}{PQ} = \frac{y}{R} \text{ as } NN' = PQ$$

$$\Sigma = \frac{y}{R}$$

$$\therefore E = \frac{\sigma}{\Sigma}$$

$$\Rightarrow \sigma = \Sigma \cdot E = \frac{y}{R} \cdot E$$

$$\therefore \frac{\sigma}{y} = \frac{E}{R}$$

...(1)

$$y \times \frac{E}{A} \times dR \times y = \frac{E}{R} y^2 \cdot dA$$

$$\therefore M = \int \frac{E}{R} y^2 \cdot dA = \frac{E}{R} \int y^2 \cdot dA$$

$$\therefore I = \int y^2 \cdot dA$$

$$\therefore M = \frac{E}{R} \cdot I \quad \dots(2)$$

compare (1) and (2) we can write

$$\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}$$

Assumption :

(i) The material is homogenous, isotropic and has same value of modulus of elasticity for compression and tension.

(ii) The beam material is stressed within the elastic limit.

(iii) The transverse cross section remain plane and perpendicular to neutral surface after bending.

(iv) The beams is initially straight and all longitudinal filaments bend into circular arcs with a common centre of a curvature.

(v) The plane of loading must contain a principal axis of the beam cross section and the loads must be perpendicular to the longitudinal axis of the beam.

Q. 5. (e) Determine the dimensions of a rectangular simply supported steel beam 5 m long to carry an UDL of 10 kN/m, if the maximum permissible bending stress is 1000 N/cm². The depth of the beam is 1.5 times its width.

$$\text{Ans. } \sigma_b = 1000 \text{ N/cm}^2 = 10,000 \text{ kN/m}^2$$

$$M_b = 25 \times 2.5 - 10 \times \frac{2.5}{2} \times \frac{2.5}{2} = 3125 \text{ kN/m}$$

$$\frac{M_b}{I} = \frac{\sigma_b}{y}$$

$$\therefore \frac{3125}{\frac{bd^3}{12}} = \frac{10,000}{d/2}$$

$$d = 1.5b$$

$$\therefore \frac{3125}{\frac{b(1.5b)^3}{12}} = \frac{10,000}{\left(\frac{1.5b}{2} \right)}$$

$$\therefore b = 2027 \text{ cm (width)}$$

$$\text{and } d = 30.41 \text{ cm (depth)}$$

Q. 5. (f) Design a circular solid shaft to transmit 80 kW power at 200 rpm, if the twist in the shaft is not to exceed 2° in 3m length of the shaft and maximum shear stress is limited to 70 MN/m².

$$\text{Take mod. of rigidity } G = 90 \text{ GN/m}^2$$

$$\text{Ans. } P = 80 \text{ kW}$$

$$N = 200 \text{ rpm}$$

$$\theta = 2^\circ = 628 \text{ rad}$$

$$l = 3 \text{ m}$$

$$P = \frac{2\pi NT}{60 \times 1000}$$

$$80 = \frac{2\pi \times 200 \times T}{60 \times 1000} \therefore T = 3819.72 \text{ N-m}$$

$$T = \frac{\pi}{16} \tau d^3$$

$$3819.72 = \frac{\pi}{16} \times 70 \times 10^6 d^3$$

$$d = 65.26 \text{ mm}$$

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\frac{3819.72}{\frac{\pi}{32} d^4} = \frac{90 \times 10^9 \times 6.28}{3}$$

$$d = 78.38 \text{ mm}$$

Larger value of diameter is safe value $\therefore d = 78.38 \text{ mm}$